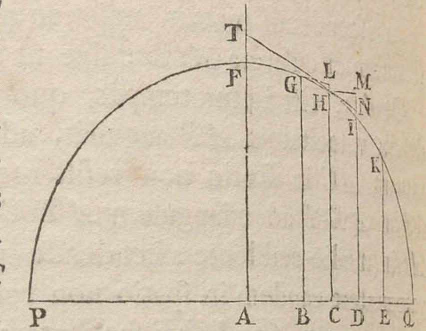
PROPOSITIO X. PROBLEMA III.

Tendat uniformis vis gravitatis directe ad planum horizontis, ſitque reſiſtentia ut medii denſitas & quadratum velocitatis conjunctim: requiritur tum medii denſitas in locis ſingulis, quæ faciat ut corpus in data quavis linea curva moveatur; tum corporis velocitas & medii reſiſtentia in locis ſingulis.

Sit PQ planum illud plano ſchematis perpendiculare; $PFHQ$ linea curva plano huic occurrens in punctis P & Q; G, H, I, K loca quatuor corporis in hac curva ab F ad Q pergentis; & GB, HC, ID, KE ordinatæ quatuor parallelæ ab his punctis ad horizontem demiſſæ, & lineæ horizontali PQ ad puncta B, C, D, E inſiſtentes; & ſint BC, CD, DE diſtantiæ ordinatarum inter ſe æquales. A punctis G & H ducantur rectæ GL, HN curvam tangentes in G & H, & ordinatis CH, DI ſurſum productis occurrentes in L & N, & compleatur parallelogrammum $HCDM$. Et tempora, quibus corpus deſcribit arcus GH, HI, erunt in ſubduplicata ratione altitudinum LH, NI, quas corpus temporibus illis deſcribere poſſet, a tangentibus cadendo; & velocitates erunt ut longitudines deſcriptæ GH, HI directe & tempora inverſe. Exponantur tempora per T & t, & velocitates per $\frac{GH}{T}$ & $\frac{HI}{t}$; & decrementum velocitatis tempore t factum exponetur per $\frac{GH}{T} - \frac{HI}{t}$. Hoc decrementum oritur a reſiſtentia corpus retardante, & gravitate corpus accelerante. Gravitas, in corpore cadente & ſpatium NI cadendo deſcribente, generat velocitatem, qua duplum illud ſpatium eodem tempore deſcribi potuiſſet, ut *Galilæus* demonſtravit; id eſt, velocitatem $\frac{2NI}{t}$: at in corpore arcum HI deſcribente, auget arcum illum



ſola longitudine $HI - HN$ ſeu $\frac{MI \times NI}{HI}$; ideoque generat tantum velocitatem $\frac{2MI \times NI}{t \times HI}$. Addatur hæc velocitas ad decrementum prædictum, & habebitur decrementum velocitatis ex reſiſtentia ſola oriundum, nempe $\frac{GH}{T} - \frac{HI}{t} + \frac{2MI \times NI}{t \times HI}$. Proindeque cum gravitas eodem tempore in corpore cadente generet velocitatem $\frac{2NI}{t}$; reſiſtentia erit ad gravitatem ut $\frac{GH}{T} - \frac{HI}{t} + \frac{2MI \times NI}{t \times HI}$ ad $\frac{2NI}{t}$, ſive ut $\frac{t \times GH}{T} - HI + \frac{2MI \times NI}{HI}$ ad $2NI$.

Jam pro abſciſſis CB, CD, CE ſcribantur $-o$, o, $2o$. Pro ordinata CH ſcribatur P, & pro MI ſcribatur ſeries quælibet $Qo + Roo + So^3 +$ &c. Et ſeriei termini omnes poſt primum, nempe $Roo + So^3 +$ &c. erunt NI, & ordinatæ DI, EK, & BG erunt $P - Qo - Roo - So^3 -$ &c. $P - 2Qo - 4Roo - 8So^3 -$ &c. & $P + Qo - Roo + So^3 -$ &c. reſpective. Et quadrando differentias ordinatarum $BG - CH$ & $CH - DI$, & ad quadrata prodeuntia addendo quadrata ipſarum BC, CD, habebuntur arcuum GH, HI quadrata $oo + QQoo - 2QRo^3 +$ &c. & $oo + QQoo + 2QRo^3 +$ &c. Quorum radices $o\sqrt{1 + QQ} - \frac{QRoo}{\sqrt{1+QQ}}$, & $o\sqrt{1+QQ} + \frac{QRoo}{\sqrt{1+QQ}}$ ſunt arcus GH & HI. Præterea ſi ab ordinata CH ſubducatur ſemiſumma ordinatarum BG ac DI, & ab ordinata DI ſubducatur ſemiſumma ordinatarum CH & EK, manebunt arcuum GI & HK ſagittæ Roo & $Roo + 3So^3$. Et hæ ſunt lineolis LH & NI proportionales, ideoque in duplicata ratione temporum infinite parvorum T & t: & inde ratio $\frac{t}{T}$ eſt $\sqrt{\frac{R + 3So}{R}}$ ſeu $\frac{R + \frac{3}{2}So}{R}$; & $\frac{t \times GH}{T} - HI + \frac{2MI \times NI}{HI}$, ſubſtituendo ipſorum $\frac{t}{T}$, GH, HI, MI & NI valores jam inventos, evadit $\frac{3Soo}{2R}\sqrt{1+QQ}$. Et cum $2NI$ ſit $2Roo$, reſi-

ſtentia